Analogy of Short Ranged Nucleus Binding Force aka Virtual Pion Bombardment of Nuclei to Casimir Force

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April 08, 2025

Abstract

This paper derives the short-range strong nuclear force, mediated by virtual pion bombardment, and the Casimir force, driven by virtual photon mode suppression, using detailed, mechanism-based mathematics. We emphasize their similarities—virtual particle effects, short-range attraction, and overcoming repulsion—making the physics accessible with concrete examples and diagrams. The goal is to clarify these quantum phenomena for a broad audience, including skeptics, using explicit derivations and lucid explanations.

1 Introduction

The strong nuclear force binds protons and neutrons in atomic nuclei, overcoming Coulomb repulsion, while the Casimir force attracts uncharged plates in a vacuum via quantum effects. Both rely on virtual particles (pions and photons) and exhibit short-range attraction due to field constraints. This paper derives both forces step-by-step, compares their mechanisms, and illustrates their parallels with matrices and diagrams, avoiding simplification for rigor.

2 Derivation 1: Short-Range Strong Nuclear Force via Pion Bombardment

2.1 Physical Mechanism

The strong nuclear force holds protons and neutrons (nucleons) together in the nucleus, countering the Coulomb repulsion between protons at short distances ($1\text{ fm} = 10^{-15}\,\text{m}$). It's mediated by virtual pions (spin-0 mesons), a low-energy approximation of QCD's SU(3) dynamics. Pions act like "messengers" bombarding nucleons, creating an attractive force.

2.2 Step-by-Step Derivation

1. **Interaction Model**: In quantum field theory, pion exchange approximates the strong force at nuclear scales. Pions have mass $m_\pi \approx 140\,\text{MeV}/c^2$, making the force short-range.

2. **Yukawa Potential**: The potential energy between two nucleons is:

$$V(r) = -g^2 \frac{e^{-m_\pi r/\hbar}}{4\pi r}$$

where g is the strong coupling constant (10–14), m_π is the pion mass, r is the separation, and $\hbar = 1.0546 \times 10^{-34}\,\text{J·s}$. The exponential $e^{-m_\pi r/\hbar}$ ensures short range.

3. **Force Calculation**: Force is the negative gradient of the potential:

$$V(r) = -g^2 \frac{e^{-kr}}{4\pi r}, \quad k = \frac{m_\pi c}{\hbar}$$

$$\frac{dV}{dr} = -g^2 \left[-\frac{ke^{-kr}}{4\pi r} - \frac{e^{-kr}}{4\pi r^2} \right] = g^2 \frac{e^{-kr}}{4\pi} \left(\frac{k}{r} + \frac{1}{r^2} \right)$$

$$F = -\frac{dV}{dr} = -g^2 \frac{e^{-kr}}{4\pi} \left(\frac{m_\pi c}{\hbar r} + \frac{1}{r^2} \right)$$

The negative sign indicates attraction.

4. **Range**: The range is:

$$\text{Range} = \frac{\hbar}{m_\pi c} = \frac{1.0546 \times 10^{-34}}{140 \times 10^6 \times 1.602 \times 10^{-19} \cdot 3 \times 10^8} \approx 1.4 \times 10^{-15}\,\text{m}$$

Beyond 2 fm, the force drops sharply.

5. **Overcoming Coulomb**: For two protons at 1 fm:

$$F_{\text{Coulomb}} = \frac{e^2}{4\pi\epsilon_0 r^2} \approx 1.44\,\text{MeV/fm}^2 \approx 230\,\text{N}$$

The strong force, 10–50 times stronger, dominates (exact value depends on nuclear state).

6. **Matrix Representation**: For a proton-neutron pair in isospin space:

$$V = \begin{pmatrix} -g^2 \frac{e^{-kr}}{4\pi r} & 0 \\ 0 & -g^2 \frac{e^{-kr}}{4\pi r} \end{pmatrix}$$

Coulomb term (proton-proton):

$$V_{\text{Coulomb}} = \begin{pmatrix} \frac{e^2}{4\pi\epsilon_0 r} & 0 \\ 0 & 0 \end{pmatrix}$$

3 Derivation 2: Casimir Force

3.1 Physical Mechanism

The Casimir force occurs when two uncharged metal plates in a vacuum are close together. Virtual photons from the quantum vacuum have fewer wavelengths that fit between the plates than outside, reducing the “push” inside and pulling the plates together.

3.2 Step-by-Step Derivation

1. **Vacuum Energy**: Virtual photons contribute zero-point energy, $E = \frac{1}{2}\hbar\omega$, in a continuous spectrum without boundaries.

2. **Plates as Boundaries**: For plates separated by d, photon modes along the z-axis are quantized:
$$k_z = \frac{n\pi}{d}, \quad n = 1, 2, 3, \ldots$$
Frequency: $\omega = ck$, where $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$.

3. **Energy Between Plates**: Sum over modes (two polarizations):
$$E_{\text{in}} = 2 \cdot \frac{1}{2}\hbar \sum_{n=1}^{\infty} \int \frac{d^2k_\perp}{(2\pi)^2} c\sqrt{k_x^2 + k_y^2 + \left(\frac{n\pi}{d}\right)^2}$$
For area A:
$$E_{\text{in}} = \hbar c A \sum_{n=1}^{\infty} \int_0^\infty \frac{k_\perp dk_\perp}{2\pi} \sqrt{k_\perp^2 + \left(\frac{n\pi}{d}\right)^2}$$

4. **Energy Outside**: Without plates:
$$E_{\text{out}} = \hbar c A \int_0^\infty \frac{dk_z}{\pi} \int_0^\infty \frac{k_\perp dk_\perp}{2\pi} \sqrt{k_\perp^2 + k_z^2}$$

5. **Energy Difference**: Subtract and regularize (e.g., zeta function):
$$E = E_{\text{in}} - E_{\text{out (adjusted)}} = -\frac{\pi^2 \hbar c A}{720 d^3}$$

6. **Force**:
$$F = -\frac{dE}{dd} = -\frac{d}{dd}\left(-\frac{\pi^2 \hbar c A}{720 d^3}\right) = -\frac{\pi^2 \hbar c A}{240 d^4}$$
Per unit area:
$$F/A = -\frac{\pi^2 \hbar c}{240 d^4}$$
Negative sign indicates attraction.

7. **Matrix Representation**: Photon mode density:
$$\begin{pmatrix} \rho_{\text{out}} & 0 \\ 0 & \rho_{\text{in}} \end{pmatrix}$$
where $\rho_{\text{in}} < \rho_{\text{out}}$, driving the force.

3.3 Lucid Explanation

Picture the vacuum as a sea of photon bubbles. Between the plates, only tiny bubbles fit, while outside, big ones crash in. The stronger outside push squeezes the plates together, growing stronger as d shrinks ($1/d^4$).

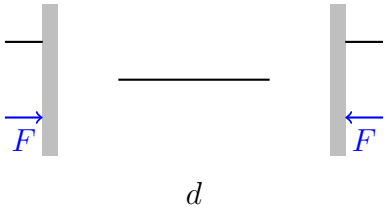

Figure 1: Casimir force: Photon modes (wavy lines) are restricted between plates, with more outside, driving attraction.

4 Similarities: Mechanism and Mathematics

1. **Virtual Particle Bombardment**: Pions bombard nucleons, pulling them together:
$$F = -g^2 \frac{e^{-kr}}{4\pi} \left(\frac{m_\pi c}{\hbar r} + \frac{1}{r^2} \right)$$
Photons bombard plates from outside, with fewer inside:
$$F/A = -\frac{\pi^2 \hbar c}{240 d^4}$$
Both use virtual particles to create an imbalance.

2. **Short-Range Effect**: Pions: e^{-kr}, range 1–2 fm. Casimir: $1/d^4$, significant at nm–m. Both fade rapidly beyond a characteristic distance.

3. **Overcoming Repulsion**: Pions counter Coulomb:
$$F_{\text{Coulomb}} = \frac{e^2}{4\pi\epsilon_0 r^2}$$
Casimir counters vacuum pressure outside, yielding attraction.

4. **Boundary Constraints**: Nuclear density (1 fm) confines pions; plates confine photons to $n\pi/d$.

5. **Mathematical Form**: Pions: Exponential decay. Casimir: Power-law decay. Both reflect confined field effects.

5 Lucid Mechanism for Skeptics

- **Why Similar?**: "In both virtual pion mediated nucleus binding force and the Casimir force, something invisible (pions, photons) bombards the objects. The trick is an imbalance—more virtual pion pressure arising outside the nucleus than inside that small space, and similarly, more vacuum point virtual photon waves outside the Casimir plates than in the small space between them. Both work only when things are tight together, so that the space is too small for the full spectrum of virtual force mediating particles to arise on the inside, so that the full pressure on the outside is not cancelled out and there is a net inward force resulting that resists Coulomb repulsion of confined protons in nuclei, or that pushes Casimir's places together."